Supersymmetry, Local Horizontal Unification, and a Solution to the Flavor Puzzle

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Abstract

Supersymmetric gauge models with local horizontal symmetries are known to generate large flavor changing neutral current effects induced by supersymmetry breaking D–terms. We show how the presence of a $U(1)$ gauge symmetry solves this problem. We then construct a realistic gauge model with $SU(2)_H \times U(1)_H$ as the local horizontal symmetry and suggest that the $U(1)_H$ factor may be identified with the anomalous $U(1)$ induced by string compactification. This model explains the observed hierarchies among the quark masses and mixing angles, accommodates naturally the solar and atmospheric neutrino data, and provides simultaneously a solution to the supersymmetric flavor problem. The model can be excluded if the rare decay $\mu \to e\gamma$ is not observed in the current round of experiments.

I. INTRODUCTION

One of the fundamental puzzles of the standard model is a lack of understanding of the fermion mass and mixing hierarchies. A promising approach to resolve this puzzle is to use horizontal symmetries, either global and local, that transform fermions of one generation into another [1]. The fact that in the limit of vanishing Yukawa couplings, the standard model has an enormous $[SU(3)_L^q \times SU(3)_R^u \times SU(3)_R^d \times SU(3)_L^\ell \times SU(3)_R^e \times U(1)_{B-L}]$ horizontal symmetry makes this approach quite plausible.

In this paper we shall be concerned with *local* realization of horizontal symmetries. Local symmetries have certain clear–cut advantages over their global counterparts. The most significant difference is perhaps the strong suspicion that global (but not local) symmetries are susceptible to explicit violation through quantum gravitational effects.

The presence of supersymmetry appears to put additional constraints on models with *local* horizontal symmetries [2]. For instance, if one chooses a general simple non–Abelian group G as the local horizontal symmetry, in the presence of supersymmetry breaking, the D–terms associated with G will induce a non–negligible splitting among the slepton and squark masses of different generations. In turn they induce unacceptable amount of

flavor changing neutral current effects. Since the squark mass splittings are independent of the horizontal gauge couplings as long as the horizontal group is simple, there is no free parameter that can be used to dial down these splittings to an acceptable level. This poses a serious roadblock to the use of local horizontal symmetries as a way to understand the fermion mass hierarchies in a supersymmetric context.

If the D–term mass splittings between squarks (or sleptons) were absent (or if they were under control), local horizontal symmetries can neatly address the SUSY flavor problem that plagues the generic softly broken supersymmetric standard model. Such a solution would be highly desirable especially in scenarios where supersymmetry breaking is communicated to the MSSM sector through supergravity. The clear advantage is that no special assumption need be made about the Kahler potential (or the superpotential), apart from the requirement of gauge invariance. Two different approaches have been adopted in the past that evade the aforementioned D–term difficulty: (i) assume the horizontal symmetry to be *global* [3], or (ii) use a discrete gauge symmetry [4]. In both cases there are no associated D–terms.

In this paper we propose a solution to the D–term problem that would facilitate the use of *true gauge symmetries* to address simultaneously the fermion mass problem and the supersymmetric flavor problem[1]. We show that by adjoining a local $U(1)$ symmetry to the existing non–Abelian horizontal symmetry G, the mass splittings between different generations of squarks caused by the D–terms can be brought under control. The mass splittings will now depend quadratically on the ratio of the two gauge couplings, which can be adjusted to make the FCNC effects sufficiently small. We illustrate this mechanism by using the example of a horizontal $SU(2)_H \times U(1)_H$ model. We then construct a fully realistic model using this group and show how the $U(1)$ factor may be identified with the anomalous $U(1)$ that arises in superstring compactification. We find this model to be quite predictive in the neutrino as well as in the quark and lepton sectors. Specifically, it supports the large angle atmospheric and small angle solar neutrino oscillations. The model can be excluded if the rare decay $\mu \to e\gamma$ is not observed in the current round of experiments. Furthermore, this model can easily be grand unified into an $SU(5)$ or $SO(10)$ group.

II. SUPPRESSION OF D–TERM SPLITTINGS IN THE PRESENCE OF AN EXTRA $U(1)$

Let us consider the gauge group of the theory to be $SU(3)_C \times SU(2)_L \times U(1)_Y \times G_H$, where the horizontal group G_H is chosen to be $SU(2)_H \times U(1)_H$ as stated before. We will choose the matter content of the model to be the same as that of the MSSM with the addition of three right–handed neutrinos (denoted by ν_i^c). These ν_i^c fields are required for the cancellation of triangle and global $SU(2)_H$ anomalies. An immediate consequence is that the left–handed neutrinos will have small masses induced by the seesaw mechanism. The horizontal quantum numbers are chosen under the straightforward assumption that the particles of the first two generation belong to doublets of the $SU(2)_H$ group whereas the particles of the third generation are in the singlet representation. (This assignment is same

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[1]For a specific gauged $SO(3)$ model with symmetry breaking via fundamental fields, see Ref. [5].

as in Ref. [3]). Furthermore, we impose the unifiability condition, which means that all 16 members of a generation have the same horizontal charges. Thus the matter content of the model in the standard notation is:

$$
\begin{aligned}
\{Q_a,\ L_a,\ u_a^c,\ d_a^c,\ e_a^c,\ \nu_a^c\} &: \mathbf{2}(-1) \\
\{Q_3,\ L_3,\ u_3^c,\ d_3^c,\ e_3^c,\ \nu_a^3\} &: \mathbf{1}(0)
\end{aligned}
\tag{1}
$$

where we have exhibited the $SU(2)_H \times U(1)_H$ quantum numbers. Here $a = 1 - 2$ is the $SU(2)_H$ index. The Higgs sector consists of the following fields:

$$
\{H_u,\ H_d\} : \mathbf{1}(0); \quad \phi_a : \mathbf{2}(1); \quad \overline{\phi}_a : \mathbf{2}(-1); \quad \chi : \mathbf{1}(1); \quad \overline{\chi} : \mathbf{1}(-1); \quad S_i : \mathbf{1}(0) \ (i = 1 - 2) \ . \tag{2}
$$

Here H_u, H_d are the usual MSSM doublet fields, while all the other fields are singlets of the standard model. Given the horizontal symmetry group G, this Higgs system is the minimal choice that can properly break G without breaking supersymmetry. The $(\phi, \overline{\phi})$ fields break $SU(2)_H$ completely, while the $(\chi, \overline{\chi})$ fields break $U(1)_H$. The S_i fields are necessary to allow cubic terms in the superpotential, a requirement if the horizontal gauge symmetry is to be broken in the supersymmetric limit with only renormalizble terms.

Let us first demonstrate how the suppression of the D–term mass splittings between different generation squarks (and sleptons) arises in this model. In this particular model the problem concerns the $SU(2)_H$ D–terms which can potentially split the masses of the first two generations. Note that the $U(1)_H$ D–term will not induce mass splittings within the first two generations. The most general superpotential involving the Higgs fields has the form:

$$
W = \mu_\phi \phi \overline{\phi} + \lambda \phi \overline{\phi} S_1 + W'(\chi \overline{\chi}, S_i) \ , \tag{3}
$$

where we wont need the explicit form of the piece W' involving $(\chi \overline{\chi}, S_i)$ fields. In the supersymmetric limit, we have $\langle \phi \rangle = \langle \overline{\phi} \rangle \equiv V_\phi$ and $\langle \chi \rangle = \langle \overline{\chi} \rangle \equiv V_\chi$. We shall make the reasonable assumption that the scale of horizontal symmetry breaking (V_ϕ, V_χ) is much greater than the scale of soft supersymmetry breaking terms, M_{SUSY}. The requirement of vanishing F–terms in the supersymmetric limit implies $F_\phi = (\mu_\phi + \lambda S_1)\overline{\phi} = 0$ and $F_S = \lambda \phi \overline{\phi} + \partial W'/\partial S_1 = 0$. Including arbitrary soft supersymmetry breaking, the scalar potential involving $(\phi, \overline{\phi})$ fields is given by:

$$
\begin{aligned}
V &= |\mu_\phi + \lambda S|^2 \left(|\phi|^2 + |\overline{\phi}|^2\right) + \left|\lambda \phi \overline{\phi} + \partial W'/\partial S\right|^2 + \frac{1}{8}(g_{2H}^2 + 4g_{1H}^2)(|\phi|^2 - |\overline{\phi}|^2)^2 \\
&\quad + m_\phi^2 |\phi|^2 + m_{\overline{\phi}}^2 |\overline{\phi}|^2 + \{B_\phi \mu_\phi \phi \overline{\phi} + A_\phi \lambda \phi \overline{\phi} S + H.c.\} \ .
\end{aligned}
\tag{4}
$$

The coefficients in the last line of Eq. (4) $(m_\phi^2, m_{\overline{\phi}}^2, B_\phi^2, A_\phi^2)$ are all of order M_{SUSY}^2. Here we have adopted the point of view that in the absence of a symmetry, the superpartner masses at the Planck scale should be arbitrary. Specifically, we do not assume universality of scalar masses.

Minimizing this potential (Eq. (4)) with respect of ϕ and $\overline{\phi}$ fields and subtracting the two extremization conditions, we arrive at the relation:

$$
(|\overline{\phi}|^2 - |\phi|^2)\left[\frac{\lambda^*}{\phi \overline{\phi}}(\lambda \phi \overline{\phi} + \partial W'/\partial S) - \frac{1}{4}(g_{2H}^2 + 4g_{1H}^2) + B_\phi \mu_\phi + A_\phi \lambda S\right] = (m_{\overline{\phi}}^2 - m_\phi^2) \ . \tag{5}
$$

Noting that $\lambda\phi\overline{\phi} + \partial W'/\partial S_1 = \mathcal{O}(M_{\mathrm{SUSY}}V_\phi)$, Eq. (5) implies:

$$\frac{1}{4}(g_{2H}^2 + 4g_{1H}^2)(|\phi|^2 - |\overline{\phi}|^2) \simeq -(m_\phi^2 - m_{\overline{\phi}}^2) + \mathcal{O}(M_{\mathrm{SUSY}}/V_\phi)M_{\mathrm{SUSY}}^2 \ . \tag{6}$$

The contribution to the squark (or slepton) mass splittings from the D–term is given by:

$$\Delta m_{\tilde{q}}^2 \simeq \frac{1}{4}g_{2H}^2(|\phi|^2 - |\overline{\phi}|^2) \simeq \frac{g_{2H}^2}{g_{2H}^2 + 4g_{1H}^2}(m_{\overline{\phi}}^2 - m_\phi^2) \ . \tag{7}$$

Note that in the absence of the $U(1)_H$ group (i.e., if $g_{1H} = 0$), $\Delta m_{\tilde{q}}^2 \simeq m_{\overline{\phi}}^2 - m_\phi^2$ which is independent of the gauge coupling and is arbitrary (i.e., anywhere between $(100\ \mathrm{GeV})^2 - (1000\ \mathrm{GeV})^2$. Although these D–terms contribute to diagonal squark masses, because they are not universal, once Cabibbo rotation on the quark fields are made, they do contribute to flavor changing processes. The most stringent constraints arise from $K^0 - \bar{K}^0$ mixing and the rare decay $\mu \to e\gamma$. The $K_L - K_S$ mass difference sets a constraint [6] (from the most stringent $(LL)(RR)$ operator) $[\Delta m_{\tilde{q}}^2/m_{\tilde{q}}^2]\theta_C \leq 1 \times 10^{-3}(m_{\tilde{q}}/500\ GeV)$, where $m_{\tilde{q}}$ denotes the average squark mass. The constraint arising from $\mu \to e\gamma$ is similar. Clearly, if $g_{1H} \to 0$, the D–term splittings will grossly contradict experiments[2] if the squark masses are below a TeV. On the other hand, in the presence of the extra gauge coupling g_{1H}, we can control the FCNC processes to adequate levels. For example, if $g_{2H}/g_{1H} = (1/3 - 1/7)$, which is not at all unreasonable, then $\Delta m_{\tilde{q}}^2 \simeq (1/40 - 1/200)(m_{\overline{\phi}}^2 - m_\phi^2)$. For $m_{\tilde{q}} \sim (300 - 500)$ GeV, this is comfortably consistent with experiments. Note that the soft supersymmetry breaking mass terms m_ϕ^2 and $m_{\overline{\phi}}^2$ do not run below the horizontal scale (since the ϕ and $\overline{\phi}$ fields have masses of order V_ϕ). On the other hand, the masses of the squarks do run below V_ϕ and in this process receive a flavor universal contribution from the gauginos. For comparable values of initial (Planck scale) gaugino and squark masses, the factor $m_\phi^2/m_{\tilde{q}}^2$ is suppressed by about $\sim 1/10$ because of the running.

While we used a specific example to illustrate the proposed mechanism to cure the D–term problem, its features prevail in more general contexts. For example, we could use alternative superpotentials (Cf. Eq. (3)) such as $W = \lambda S_1(\phi\overline{\phi} - \mu^2) + W'$ or one involving non–renormalizable operators. The FCNC problem in the absence of $U(1)_H$, and its solution via $U(1)_H$ will be identical to the case discussed above.

III. A REALISTIC MODEL OF FERMION MASS AND MIXING HIERARCHIES

We will now show that the model presented in the previous section can naturally explain the fermion mass and mixing hierarchies. We take the viewpoint that all operators consistent with gauge invariance are allowed in the Lagrangian. This includes non–renormalizable operators, which will be suppressed by appropriate inverse powers of the Planck mass. The coefficients of such operators will all be assumed to be of order unity.

Let us first consider the quark sector of the theory. It is easy to see that the superpotential consistent with $SU(2)_H \times U(1)_H$ symmetry is given by:

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[2]For an exception where θ_C arises almost entirely from the up–quark sector, see Ref. [7].

$$W_{\text{Yuk}} = h_{33}^u Q_3 u_3^c H_u + h_{33}^d Q_3 d_3^c H_d + \frac{h_{23}^u}{M} \epsilon^{ab} Q_a u_3^c H_u \phi_b + \frac{h_{23}^d}{M} \epsilon^{ab} Q_a d_3^c H_d \phi_b + \frac{h_{32}^u}{M} \epsilon^{ab} Q_3 u_a^c H_u \phi_b$$

$$+ \frac{h_{32}^d}{M} \epsilon^{ab} Q_3 d_a^c H_d \phi_b + \frac{h_{22}^u}{M^2} Q_a u_b^c H_u \phi_p \phi_q \epsilon^{ap} \epsilon^{bq} + \frac{h_{22}^d}{M^2} Q_a d_b^c H_d \phi_p \phi_q \epsilon^{ap} \epsilon^{bq}$$

$$+ \frac{h_{12}^u}{M^2} \epsilon^{ab} Q_a u_b^c H_u \chi^2 + \frac{h_{12}^d}{M^2} \epsilon^{ab} Q_a d_b^c H_d \chi^2 \ . \tag{8}$$

Defining two small parameters $\epsilon_\phi \equiv \frac{\langle \phi \rangle}{M}$ and $\epsilon_\chi \equiv \frac{\langle \chi \rangle}{M}$, we get the following hierarchical mass matrix for both up and the down sectors:

$$M_f \;=\; v_f \begin{pmatrix} 0 & h_{12}^f \epsilon_\chi^2 & 0 \\ -h_{12}^f \epsilon_\chi^2 & h_{22}^f \epsilon_\phi^2 & h_{23}^f \epsilon_\phi \\ 0 & h_{32}^f \epsilon_\phi & h_{33}^f \end{pmatrix} \tag{9}$$

with $f = u, d$. The charge lepton mass matrix has an identical form as Eq. (9), as does the Dirac neutrino mass matrix (identify $f = \ell, \nu$ for the two cases).

The mass matrices in Eq. (9) naturally explain the fermion mass and mixing angle hierarchies. To see this in detail, assume that all the h_{ij} parameters are of order one. The mass ratios in the down–quark sector is then given by:

$$m_s/m_b \sim \epsilon_\phi^2, \quad m_d/m_s \sim \epsilon_\chi^4/\epsilon_\phi^4 \tag{10}$$

with similar results in the up–quark and the charged lepton sectors. If we choose $\epsilon_\phi \simeq 1/7$ and $\epsilon_\chi \simeq 1/20$, all the observed masses and mixing angles can be explained, with the coefficients h_{ij} taking values in the range $(1/2 - 2)$. This is a tremendous improvement over the standard model Yukawa couplings which span some six orders of magnitude. In our scheme, order one differences such as in m_μ/m_τ and m_s/m_b (the two differ by about a factor of 3 near the Planck scale) are attributed to order one differences in the h_{ij} couplings. The hierarchy m_b/m_t requires moderate to large values of $\tan\beta \sim 10 - 40$.

The zeros in the mass matrices of Eq. (9) are corrected only at very high order, and are negligible. (The (1,1) entry receives a correction of order $\epsilon_\phi^2 \epsilon_\chi^4$, the (1,3) and (3,1) entries are corrected at order $\epsilon_\phi \epsilon_\chi^2$.) The near vanishing of the (1,1) entry, along with the (anti)-symmetry of the (1,2) entry leads to a successful quantitative prediction for the Cabibbo angle; the vanishing of the (1,3) and (3,1) entries yield a relation for V_{ub} (and V_{td}):

$$|V_{us}| \simeq \left| \sqrt{m_d/m_s} - \sqrt{m_u/m_c} e^{i\alpha} \right| \ ; \quad |V_{ub}|/|V_{cb}| \simeq \sqrt{m_u/m_c} \ ; \quad |V_{td}|/|V_{ts}| \simeq \sqrt{m_d/m_s} \ . \tag{11}$$

The last two relations [8] could serve as future tests of the model.

Turning now to the leptonic sector, as noted, the charged lepton and the Dirac neutrino mass matrices have identical forms as Eq. (9). The ν_i^c Majorana mass matrix is obtained from the Lagrangian:

$$\mathcal{L}^{\nu^c} = f_{33} \nu_3^c \nu_3^c \Delta + \frac{f_{23}}{M} \epsilon^{ab} \nu_a^c \nu_3^c \Delta \phi_b + \frac{f_{22}}{M^2} \nu_a^c \nu_b^c \Delta \phi_p \phi_q \epsilon^{ap} \epsilon^{bq}$$

$$+ \frac{f_{13}}{M^3} \epsilon^{ab} \nu_a^c \nu_3^c \Delta \overline{\phi}_b \chi^2 + \frac{f_{12}}{M^4} \nu_a^c \nu_b^c \Delta \overline{\phi}_p \overline{\phi}_q \epsilon^{ap} \epsilon^{bq} \chi^2 + \frac{f_{11}}{M^6} \nu_a^c \nu_b^c \phi_p \overline{\phi}_q \epsilon^{ap} \epsilon^{bq} \chi^4 \ . \tag{12}$$

At the level of the standard model, a bare mass term will be allowed for the ν_3^c Majorana mass. We have assumed it to arise from the VEV of a field Δ that breaks $B - L$ symmetry. When the model is embedded into a left–right symmetric or $SO(10)$ framework, or if the $B - L$ symmetry of the model as it stands is gauged, the Majorana masses of the ν^c fields will require the VEV of such a multiplet. Apart from the motivation to unify, we follow this path since then R–parity violating terms will be automatically eliminated from the Lagrangian.

Due to the intricacy of the seesaw diagonalization, we have kept the lowest non–vanishing terms in all entries of the Majorana ν^c matrix, which is given by:

$$
M_{\nu^c} \;=\; \langle\Delta\rangle \begin{pmatrix} f_{11}\epsilon_\phi^2\epsilon_\chi^4 & f_{12}\epsilon_\phi^2\epsilon_\chi^2 & f_{13}\epsilon_\phi\epsilon_\chi^2 \\ f_{12}\epsilon_\phi^2\epsilon_\chi^2 & f_{22}\epsilon_\phi^2 & f_{23}\epsilon_\phi \\ f_{13}\epsilon_\phi\epsilon_\chi^2 & f_{23}\epsilon_\phi & f_{33} \end{pmatrix} \; . \tag{13}
$$

Using the seesaw formula the light left–handed neutrino mass matrix is obtained to be:

$$
M_\nu^{\text{light}} \;=\; -\frac{v_u^2}{\langle\Delta\rangle} \begin{pmatrix} F_{11}\epsilon_\chi^4/\epsilon_\phi^2 & F_{12}\epsilon_\chi^2/\epsilon_\phi^2 & F_{13}\epsilon_\chi^2/\epsilon_\phi \\ F_{12}\epsilon_\chi^2/\epsilon_\phi^2 & F_{22}\epsilon_\phi^{-2} & F_{23}\epsilon_\phi^{-1} \\ F_{13}\epsilon_\chi^2/\epsilon_\phi & F_{23}\epsilon_\phi^{-1} & F_{33} \end{pmatrix} \; . \tag{14}
$$

Here F_{ij} are functions of various combinations of h_{ij}^ν and f_{ij}, and are expected to be of order one.

It is amusing to note that the largest entry in Eq. (14) corresponds to the mass of ν_μ. The light neutrino mass hierarchy predicted in the model is $m_{\nu_e} \ll m_{\nu_\tau} \ll m_{\nu_\mu}$. If we set $\langle\Delta\rangle = 2 \times 10^{16}$ GeV, which is the supersymmetric unification scale, $m_{\nu_\mu} \approx 7 \times 10^{-2}$ eV. This is in the right range to explain the atmospheric neutrino data via $\nu_\mu - \nu_\tau$ oscillations. The relevant mixing angle is given by $\theta_{\mu\tau}^{\text{osc}} \simeq (F_{23}/F_{22} - h_{23}^\ell/h_{33}^\ell)\epsilon_\phi$. This angle can be near maximal (45^0) if, for example, $F_{23}/F_{22} \sim 3 - 4$. This appears quite plausible, given that F_{ij} are non–trivial combinations of the Yukawa couplings. $\{F_{23}/F_{22} = (f_{13}f_{22}h_{33}^\nu + f_{12}f_{23}h_{33}^\nu - f_{12}f_{33}h_{32}^\nu + f_{13}f_{23}h_{32}^\nu)/[(h_{12}^\nu(f_{23}^2 - f_{22}f_{33})]\}$. As for the solar neutrino problem, it is explained by small angle $\nu_e - \nu_\tau$ MSW oscillations. The mass of ν_τ is of order $\epsilon_\phi^2 \times m_{\nu_\mu} \sim 10^{-3}$ eV. The $\nu_e - \nu_\tau$ mixing angle is of order $\epsilon_\chi^2/\epsilon_\phi \simeq 0.02$. Both parameters neatly fit the desired values [9]. Large angle ν_e oscillations are very unlikely in this model, if it is established the model could be excluded.

IV. SOLVING THE SUPERSYMMETRIC FLAVOR PROBLEM

The model presented here has a built–in solution to the supersymmetric flavor problem. In fact, part of our motivation to use the horizontal symmetry was to address this problem. Since the horizontal symmetry is local, no explicit violation is expected from quantum gravitational effects. As already discussed (see Sec. II), augmenting the horizontal symmetry group by a $U(1)$ factor alleviates flavor violation arising from the horizontal $SU(2)_H$ D–terms. Flavor violation in the squark (and the slepton) sector will however be induced, once effects of horizontal symmetry breaking are included. We will now show that such violations are not excessive and are consistent with present FCNC constraints.

We assume that supersymmetry breaking occurs in a hidden sector and is communicated to the MSSM sector by supergravity. However, we do not make any special assumption about

the Kahler potential or the superpotential. In particular, we do not assume that the scalar masses are universal or that the supersymmetry breaking trilinear A–terms are proportional to the superpotential Yukawa couplings. The soft scalar masses arise from general Kahler potential terms. For the first two generation squarks the dominant contribution arises from

$$\mathcal{L} = \int d^4\theta Q_a^\dagger Q_a \frac{z^* z}{M_{\text{Planck}}^2} \qquad (15)$$

where z is a hidden sector (spurion) field with nonzero F–component that breaks supersymmetry. Identifying $F_z^2/M_{\text{Planck}}^2 \equiv M_{\text{SUSY}}^2$, we see that the dominant masses for the first two generations are universal as a consequence of the $SU(2)_H$ horizontal symmetry. Non–universal corrections will arise from terms such as

$$\mathcal{L} = \int d^4\theta (Q_a^\dagger \phi_a)(\phi_b^\dagger Q_b) \frac{z^* z}{M_{\text{Planck}}^4} \qquad (16)$$

and a similar term with ϕ is replaced by $\overline{\phi}$. Compared to the dominant contribution (Eq. (15), these non–universal terms are suppressed by a factor $\epsilon_\phi^2 \sim 2 \times 10^{-2}$. Since these corrections contribute to diagonal entries in the squark mass matrix, any FCNC effect will require an additional quark mixing angle ($\sim \theta_C \sim \sqrt{m_d/m_s} \simeq 0.2$). Furthermore, as noted earlier, these non–universal corrections get diluted by about a factor of $1/10$ in the squark sector through RGE effects proportional to the gaugino (mainly the gluino) mass. It is convenient to define a set of parameters δ_{12}^d as the ratio of the (1,2) entry of the squark mass matrix to the average squark mass–squared in a basis where the quark fields are physical [6]. We then estimate $(\delta_{12}^d)_{LL,RR} \sim (2 \times 10^{-2}) \times (0.2) \times (0.1) = 4 \times 10^{-4}$. This is to be compared with the experimental limit on this quantity, $(\delta_{12}^d)_{LL,RR} \leq 1 \times 10^{-3}$ valid for an average squark mass of 500 GeV [6]. We see broad agreement with experiment.

Analogous discussion in the first two generation slepton sector leads to a prediction $(\delta_{12}^\ell)_{LL,RR} \simeq \epsilon_\phi^2 \theta_{e\mu} \simeq (2 \times 10^{-2}) \times (0.07) \simeq 1.4 \times 10^{-3}$. Here we have taken the $e - \mu$ mixing angle to be $\sqrt{m_e/m_\mu} \simeq 0.07$, appropriate to the mass matrix of Eq. (9). Note that unlike the squark sector, there is no significant dilution effect due to the RGE (since sleptons are color neutral). This number should be compared with the constraint from the present experimental limit on $\mu \to e\gamma$, which is $(\delta_{12}^\ell)_{LL,RR} \leq (4.0 \times 10^{-3} - 1.8 \times 10^{-2})$ corresponding to $m_{\tilde{\ell}} \simeq 100$ GeV and for $x \equiv m_{\tilde{\gamma}}^2/m_{\tilde{\ell}}^2$ in the range $0.3 - 3$ [10]. Although the constraint is satisfied, the rate for $\mu \to e\gamma$ cannot be much below the present experimental limit. We estimate the rate to be at most a factor of 100 below the present limit, which will soon be tested.

As for the supersymmetry breaking trilinear A terms, they arise in supergravity models from superpotential terms such as

$$\mathcal{L} = \int d^2\theta Q_3 d_3^c H_d \frac{z}{M_{\text{Planck}}} \ . \qquad (17)$$

The resulting coefficients of the trilinear scalar terms are of order M_{SUSY}. Horizontal gauge invariance implies that in our model, the structure of the A–terms is identical to that of the superpotential in Eq. (8). However, the coefficients of the A matrix are not proportional to the Yukawa matrix arising from Eq. (8). This non–proportionality will lead to FCNC

processes. We estimate the parameter $(\delta_{12}^d)_{LR} \sim \epsilon_\chi^2 A v_d / m_{\tilde{q}}^2$. Choosing $m_{\tilde{q}} = 500$ GeV and $\tan\beta = 30$, we obtain $(\delta_{12}^d)_{LR} \simeq 3 \times 10^{-5}$, which is well below the experimental limit on this quantity arising from $K^0 - \bar{K}^0$ mixing $((\delta_{12}^d)_{LR} \le 3 \times 10^{-3})$. A similar estimate (perhaps slightly smaller value, since $\sqrt{m_e/m_\mu} \simeq (1/3)\sqrt{m_d/m_s}$) will hold for the leptonic $(\delta_{12}^\ell)_{LR,RL}$. For slepton masses of 500 GeV, the constraint from $\mu \to e\gamma$ is $(\delta_{12}^\ell)_{LR,RL} \le 2 \times 10^{-5}$. We see that the constraint is quite tight. Allowing for unknown order one coefficients (or some proportionality of the A terms) we conclude that $\mu \to e\gamma$ cannot be much below the present experimental limit. Since the coefficients of $\tilde{\mu}_L \tilde{e}_R$ and $\tilde{\mu}_R \tilde{e}_L$ are approximately the same, we expect that both helicity muons will participate in the decay, unlike the grand unification effects discussed in Ref. [11] where the decay $\mu_L \to e_R\gamma$ is suppressed. The rate for the decay $\tau \to \mu\gamma$ is estimated to be two orders of magnitude below the present limits.

V. COMMENTS AND CONCLUSIONS

Before concluding a few remarks are in order.

(i) An important check for the renormalizability of the model is that the horizontal gauge symmetries be anomaly free. $SU(2)_H$ is automatically free of chiral anomalies. Cancellations of anomalies involving the $U(1)_H$ should however be ensured. With the assignment of $U(1)_H$ charges as given, we find the anomaly coefficients for $U(1)_H \times [SU(2)_L]^2$, $U(1)_H \times [SU(3)_C]^2$, $U(1)_H \times [SU(2)_H]^2$ and $U(1)_H \times [U(1)_Y]^2$ are all equal to -8. (We have used the GUT normalization for the Y quantum number.) It is then very tempting to identify the $U(1)_H$ as the pseudo–anomalous $U(1)$ that arises in superstring compactification [12]. An attractive aspect of this identification is that the χ and $\bar{\chi}$ fields can play the role of the singlet fields for the purpose of model building [13] and the dominant mode of supersymmetry breaking could be via the D–terms of the anomalous $U(1)$ group. In that case, one of them (which can be chosen to be the χ field) picks up a VEV of order $\frac{1}{10}M_{\mathrm{Planck}}$. Thus if we scale all higher dimensional operators by the Planck mass, we get the desired order for the ϵ_χ. It is then clear that we must choose $\langle\phi\rangle \simeq \langle\bar{\phi}\rangle$ also of the same order. This identification will go well with the fits obtained from the fermion masses. The $\bar{\chi}$ could be used to break supersymmetry if we kept only the $\chi\bar{\chi}$ term in the superpotential as in Ref. [13]. We do not pursue this line here. It might be mentioned that anomaly cancellation can also occur by introducing fields which are vector–like under the standard model, but chiral under the $U(1)_H$.

(ii) The model as it stands can easily be embedded into the grand unification groups such as $SU(5)$ or $SO(10)$ since the horizontal quantum numbers are common to all the quarks and leptons that fit into a single multiplet of the above groups. As usual, for the case of $SU(5)$ unification the MSSM doublets must be embedded into **5** and $\bar{\mathbf{5}}$ representations of $SU(5)$. For the case of $SO(10)$ grand unification, additional multiplets belonging to **126** or **16** will be needed.

(iii) CP violation in the model can arise via complex Yukawa couplings as in the standard model. There are additional supersymmetric source for CP violation. An interesting possibility is that ϵ'/ϵ in the K meson system can be explained through the gluino penguin. As noted earlier, the parameter $(\delta_{12}^d)_{LR,RL} \simeq 3 \times 10^{-5}$ in our model. If its argument is of order one, it is of the right magnitude to explain $\epsilon'/\epsilon \simeq 2.7 \times 10^{-3}$ [14]. $(\delta_{11}^d)_{LR,RL}$,

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which arises after Cabibbo rotation, will be of order 6×10^{-6}. This will lead to a neutron (and electron) electric dipole moment very near the present experimental limit. It should, however, be noted that the horizontal symmetry by itself does not fully cure the electric dipole moment problem, since the B parameter and the gaugino masses should have small phases. An interesting scenario would be one where SUSY breaking terms arising through the superpotential only (the A–terms) have order one phases.

In conclusion, we have discussed a way to avoid excessive FCNC effects induced through D–term mass splittings between squarks and sleptons of different generations in models with local horizontal symmetries. We have constructed an anomaly free model with $SU(2)_H \times U(1)_H$ as the local horizontal symmetry group and shown that it can lead to a proper understanding of the observed hierarchies among the quark and lepton masses and their mixings. We have shown that the model provides a simultaneous solution to the supersymmetric flavor problem. Without any additional assumption this model also leads to a desirable pattern of neutrino masses and mixings; it supports small angle oscillations for the solar and large angle oscillations for the atmospheric neutrino data. Although flavor violation in the model is under control, it is not unobservable. The rare decay $\mu \to e\gamma$ is predicted to be near the present experimental limit.

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